# Synthesis and Dynamic Simulation of an Offset Slider-Crank Mechanism 

Julius Thaddaeus


#### Abstract

Slider-crank mechanism plays a significant role in the mechanical manufacturing areas. The slider crank mechanism is a particular four-bar mechanism that exhibits both linear and rotational motion simultaneously. It is also called four-bar linkage configurations and the analysis of four bar linkage configuration is very important. In this paper four configurations are taken into account to synthesis, simulate and analyse the offset slider crank mechanism. Mathematical formulae are derived for determining the lengths of the crank and connecting rod; the kinematic and dynamic analyses of the positions, velocities and accelerations of the links of the offset slider crank and the forces acting on them leading to sparse matrix equation to be solved using MATLAB m-function derived from the analysis; the simulation of the model in Simulink and finally, the simulation results analysis. This program solves for all the unknown parameters and displays those results in graphical forms


Keywords:Synthesis; Dynamic Simulation; Offset Slider Crank Mechanism; MATLAB; SIMULINK.

## 1. INTRODUCTION

In many situations, mechanisms are required to perform repetitive operations such as pushing parts along an assembly line and folding cardboard boxes in an automated packaging machine. Besides the above, there are other applications like a shaper machine and punching/riveting press in which the working stroke is completed under load and must be executed slowly compared to the return stroke. This results in smaller work done per unit time. A quick return motion mechanism is useful in all such applications, [1].

Quick return motion mechanisms are used on machines tools to give slow cutting stroke and a quick return stroke for a constant angular velocity of the driving crank and arc combinations of a simple linkages such as four-bar linkages and the slider crank mechanism. An inversion of the slider crank in combination with the conventional slider crank is also used. In the design of quick return mechanisms, the ratio of the crank angle for the cutting stroke to that for the return stroke is of prime
importance and it's known as the time ratio. To produce a quick return of the cutting tool, this ratio must obviously be greater than unity and as large as possible. A quick return motion mechanism is essentially a slider-crank mechanism in which the slider has different average velocities in forward and return strokes. Thus, even if the crank rotates uniformly, the slider completes one stroke quickly compared to the other stroke. An offset slider crank mechanism can be used conveniently to achieve the above objectives.

Kinematics analysis of a mechanism is one of the important and challenging problems in the context of a mechanism designing, which should be carried out in order to evaluate different aspects of a mechanism, such as the instantaneous angle displacement, angle velocity and angle acceleration or the instantaneous displacement, velocity and acceleration etc. of each component of a mechanism. There are mainly two most common approaches to accomplish kinematics analysis of a mechanism, these include, graphical method and analytical method.

The major disadvantage of the former is that the precision of the solution is very low and the data obtained out of analysing could not be further exploited. Therefore, the latter based on mathematical deducing that could achieve high precision is often employed in practice. However, one point to note is analytical method based on mathematical deducing is quite difficult and time-consuming task owing to the fact that it would require one to perform very rigorous and complicated mathematical manipulation. Therefore, in order to overcome this disadvantage, a computation software are mostly employed to fasten the computing process. In this paper, the computation function of the computation software Matlab was employed and the simulation function of Matlab/Simulink simulation platform was exploited to accomplish kinematics simulation for the offset slider-crank mechanism.

As the kinematics and dynamics simulation analysis of a mechanism is quite difficult and challenging, the forward kinematics and dynamics simulation analysis has attracted much attention from many researchers during the past decade, much work has been reported for the simulation of mechanisms, such as, S. Dutta and T. K. Naskar [6], presented a new method to design an adjustable offset slider-crank mechanism to generate a function and a path simultaneously with the lengths of the input link and the link representing offset (henceforth called offset link) varying, without any limitation on the number of precision points; Jung-Fa Hsieh, [7], used a homogenous coordinate transformation method to develop a generic mathematical model of an offset slider-crank mechanism with a translating roller-follower. Given the fundamental design parameters, the proposed methodology not only determines the pressure angle and principal curvature of the slider cam, but also generates the NC data required for machining purposes; Shrikant R. Patel and D.S. Patel, [8],
showed that the velocity ratio and force output changes with the change in height of slider. The ratio of length of slotter link to height of slider is 1.083 and at this instant the velocity ratio and force found to be with their maximum value during the stroke.

This paper demonstrates the application of theory to mechanism synthesis and analysis; formulates a dynamic simulation model of an offset slider crank mechanism using MATLAB and SIMULINK software and finally, analyses and the simulation results. In addition, the method presented in this paper can interact design with other control module. Hence, it paves underlying theoretical grounds for the optimal designing of the complex mechanism in the future.

## 2. MATHEMATICAL PROOF FOR THE CRANK AND CONNECTING ROD LENGTHS OF THE OFFSET SLIDER-CRANK

Fig. 1 shows an offset slider-crank mechanism with AB as the crank length, BC as the connecting rod length, S as the stroke distance, h as the offset height, $\beta$ as the advanced stroke angle, $\alpha$ as the return stroke angle, and $\theta_{2}$ as the crank angle.

Figure 1. Offset Slider Crank Mechanism

The extreme right-hand position $\mathrm{C}^{\prime}$ of slider is obtained when the crank $\mathrm{AB}^{\prime}$ and connecting rod $\mathrm{B}^{\prime} \mathrm{C}^{\prime}$ of the slider add up to give the farthest possible position of the slider at a distance of $(\mathrm{r}+l)$ from centre A .
$\mathrm{AC}^{\prime}=\left(\mathrm{AB}^{\prime}\right)+\left(\mathrm{B}^{\prime} \mathrm{C}^{\prime}\right)=(\mathrm{r}+\mathrm{l})$

## Distance:

$\mathrm{AC}^{\prime}=(l+\mathrm{r})$
$A C^{\prime \prime}=(l-\mathrm{r})$

Similarly the closest possible position of the slider C" is obtained when crank is at $\mathrm{B}^{\prime \prime}$ and the crank radius r is subtracts from the connecting rod length $l$ so that
$A C^{\prime \prime}=\left(\mathrm{B}^{\prime \prime} \mathrm{C}^{\prime \prime}\right)-\left(\mathrm{AB}^{\prime \prime}\right)=(\mathrm{l}-\mathrm{r})$ $\qquad$
From right-angled triangles $\mathrm{C}^{\prime \prime} \mathrm{AM}$ and $\mathrm{C}^{\prime} \mathrm{AM}$ remembering that $\angle C^{\prime \prime} A C^{\prime}=c$, we have
$\cos \angle \mathrm{MAC}^{\prime \prime}=\frac{A M}{A C^{\prime \prime}}=\frac{h}{l-\mathrm{r}}$
$\cos \angle \mathrm{MAC}^{\prime}=\frac{A M}{A C^{\prime}}=\frac{h}{l+\mathrm{r}}$
Therefore,

$$
\mathrm{C}=\cos ^{-1}\left(\frac{h}{l+\mathrm{r}}\right)-\cos ^{-1}\left(\frac{h}{l-\mathrm{r}}\right)
$$

From $\Delta A C^{\prime} C^{\prime \prime}$ in fig.2, Using Sine Rule:
Thus,
Return stroke angle $\beta=(180-\mathrm{c})$

$$
\frac{A B+B C}{\operatorname{Sin} b}=\frac{B C-A B}{\operatorname{Sin} b}=\frac{S}{\operatorname{Sin} c}
$$

And Cutting stroke angle $\alpha=(180+\mathrm{c})$
Hence, the ratio of advance to return stroke time is expressed mathematically as
$\mathrm{Q}=\frac{\text { Time of advance stroke }}{\text { Time of return stroke }}$
When Q is greater than one, the mechanism is calledQuick-

$$
\begin{equation*}
\frac{B C-A B}{\frac{h}{B+B C}}=\frac{S}{\operatorname{Sin} c} \tag{3}
\end{equation*}
$$

## return Mechanism

$$
\frac{B C^{2}-A B^{2}}{h}=\frac{S}{\sin c} \quad \text { Therefore, } \mathrm{BC}^{2}=\frac{h S}{\sin c}+\mathrm{AB}^{2}
$$

Therefore,

## Using Cosine Rule for $\triangle \mathrm{ABC}$ :

$$
\begin{aligned}
\mathrm{S}^{2} & =(\mathrm{AB}+\mathrm{BC})^{2}+(\mathrm{BC}-\mathrm{AB})^{2}-2(\mathrm{AB}+\mathrm{BC})(\mathrm{BC}-\mathrm{AB}) \operatorname{Cosc} \\
& =(\mathrm{AB}+\mathrm{BC})^{2}+(\mathrm{BC}-\mathrm{AB})^{2}-2\left(\mathrm{BC}^{2}-\mathrm{AB}^{2}\right) \operatorname{Cosc} \\
& =2 \mathrm{AB}^{2}+2 \mathrm{AB}^{2} \operatorname{Cosc}+2 \mathrm{BC}^{2}-2 \mathrm{BC}^{2} \operatorname{Cosc} \\
& =2 \mathrm{AB}^{2}(1+\operatorname{Cosc})+2 \mathrm{BC}^{2}(1-\operatorname{Cosc}) \\
\frac{s^{2}}{2} & =\mathrm{AB}^{2}(1+\operatorname{Cosc})+\mathrm{BC}^{2}(1-\operatorname{Cosc}) \\
& =\mathrm{AB}^{2}(1+\operatorname{Cosc})+\frac{h s}{\sin c}+\mathrm{AB}^{2}(1-\operatorname{Cosc}) \\
& =\mathrm{AB}^{2}(1+\operatorname{Cosc})+\mathrm{AB} 2\left(1+\operatorname{Cosc}^{2}\right)+\frac{h s}{\sin c}(1-\operatorname{Cosc}) \\
& =2 \mathrm{AB}^{2}+\frac{h s}{\sin c}(1-\operatorname{Cosc}) \\
\mathrm{AB}^{2} & =\frac{s^{2}}{4}-\frac{h s}{2 \sin c}(1-\operatorname{Cosc})
\end{aligned}
$$

Therefore,
$\overrightarrow{A B}=\left(\frac{S^{2}}{4}-\frac{h S}{2} \tan \frac{c}{2}\right)^{1 / 2}$ From trigonometry function.

Knowing that:
$\mathrm{BC}^{2}=\frac{h S}{\sin c}+\mathrm{AB}^{2}$

Now, substituting for $\mathrm{AB}^{2}$, We have:
$\mathrm{BC}^{2}=\frac{h S}{\sin c}+\frac{s^{2}}{4}-\frac{h s}{2 \operatorname{Sin} c}(1-\operatorname{Cos} \mathrm{c})$
$=\frac{h S}{\operatorname{Sin} c}-\frac{h S}{2 \operatorname{Sin} c}(1-\operatorname{Cosc})+\frac{S^{2}}{4}$

$$
=\frac{s^{2}}{4}+\frac{h s}{2}\left(1+\frac{\operatorname{Cos} c}{\operatorname{Sin} c}\right)
$$

$=\frac{S^{2}}{4}+\frac{h S}{2} \frac{1}{\tan \frac{c}{2}}$, from trigonometry function.
$\overrightarrow{B C}=\left(\frac{S^{2}}{4}+\frac{h S}{2 \tan \frac{c}{2}}\right)^{1 / 2}$

Hence, the length of Crank
$\overrightarrow{A B}=\left(\frac{S^{2}}{4}-\frac{h S}{2} \tan \frac{c}{2}\right)^{1 / 2}$

The length of the Connecting rod
$\overrightarrow{B C}=\left(\frac{S^{2}}{4}+\frac{h S}{2 \tan \frac{c}{2}}\right)^{1 / 2}$
3. THE KINEMATIC AND DYAMIC ANALYSIS LEADING TO THE SPARSE MATRIX EQUATION.
3.1 Vector Loop Equations
3.1.1. Position equations:


Figure 3. Vector Loop Schematic of the Offset Slider Crank

The schematic of a slider-crank mechanism with one degree-of-freedom is shown in Fig. 3, it is composed of four component parts, i.e., Ground $R_{1}, \operatorname{Crank} R_{2}$,

Connecting rod $R_{3}$, Offset height $R_{4}$ and
Slider C. In order to research the kinematics and dynamics of the slider-crank mechanism, a fixed reference frame ABM is attached to crank $R_{2}$ of the mechanism, as shown in Fig. 3, where origin A is the starting point of $\operatorname{crank} R_{2}$. The geometric parameters of the mechanism can be described as follows:

The link-length of crank $R_{2}$ is $\overrightarrow{A B}$ while $\overrightarrow{B C}$ for connecting $\operatorname{rod} R_{3}$ link-length, $R_{1}$ represents the position of slider C in the fixed frame, $\theta_{2}$ is the active input-angle, $\theta_{3}$ is the passive output-angle and finally, $\overrightarrow{A M}$ is for the offset $R_{4}$.

With the above-mentioned notations and with reference to Fig.3, the following closed-loop vector equation of the offset slider-crank mechanism describing the relationship between the input and the output vectors can be expressed as below:
$R_{2}+R_{3}-R_{1}-R_{4}=0$
$R_{2}+R_{3}=R_{1}+R_{4}$
$r_{2} \sin \theta_{2}+r_{3} \sin \theta_{3}-r_{1} \sin 0^{\circ}-h \sin 90^{\circ}=0$
$r_{2} \cos \theta_{2}+r_{3} \cos \theta_{3}-r_{1} \cos 0^{\circ}-r_{1} \cos 90^{\circ}=0$
Therefore, the position equations are:
$r_{2} \sin \theta_{2}+r_{3} \sin \theta_{3}-h=0$ $\qquad$
$r_{2} \cos \theta_{2}+r_{3} \cos \theta_{3}-r_{1}=0$ $\qquad$
3.1.2. Velocity Equations:

The velocity equations are obtained by differentiating the position equations:
$\frac{d \theta}{d t}=\omega$
Note, from derivatives of trigonometry functions:
$\frac{d \sin \theta}{d t}=\cos \theta$
$\frac{d \cos \theta}{d t}=-\sin \theta$

Therefore, the velocity equations are given as:

$$
\begin{equation*}
-\omega_{2} r_{2} \sin \theta_{2}-\omega_{3} r_{3} \sin \theta_{3}=\dot{r}_{1} \tag{7}
\end{equation*}
$$

$\qquad$
$\omega_{2} r_{2} \cos \theta_{2}-\omega_{3} r_{3} \cos \theta_{3}=0$. $\qquad$

Equations (7) and (8) combined to form a system of three equations with two unknowns as shown below:
$\left[\begin{array}{cc}1 & r_{3} \sin \theta_{3} \\ 0 & -r_{3} \cos \theta_{3}\end{array}\right]\left[\begin{array}{c}\dot{r}_{1} \\ \omega_{3}\end{array}\right]=\left[\begin{array}{c}-\omega_{2} r_{2} \sin \theta_{2} \\ \omega_{2} r_{2} \cos \theta_{2}\end{array}\right]$
3.1.3. Acceleration Equations:

We take the second derivatives of the position equations to obtain our acceleration equations.
$-\alpha_{2} r_{2} \sin \theta_{2}-r_{2} \omega_{2}^{2} \cos \theta_{2}-\alpha_{3} r_{3} \sin \theta_{3}-r_{3} \omega_{3}^{2} \cos \theta_{3}=\ddot{r}_{1} \ldots$. (9)
$\alpha_{2} r_{2} \cos \theta_{2}-r_{2} \omega_{2}^{2} \sin \theta_{2}+\alpha_{3} r_{3} \cos \theta_{3}-r_{3} \omega_{3}^{2} \sin \theta_{3}=0$ $\qquad$

Equations (9) and (10) also combined to form a system of three equations with two unknowns.
$\left[\begin{array}{cc}1 & r_{3} \sin \theta_{3} \\ 0 & -r_{3} \cos \theta_{3}\end{array}\right]\left[\begin{array}{l}\ddot{r}_{1} \\ \alpha_{3}\end{array}\right]=$
$\left[\begin{array}{ccc}-\alpha_{2} r_{2} \sin \theta_{2} & -r_{2} \omega_{2}^{2} \cos \theta_{2} & -r_{3} \omega_{3}^{2} \sin \theta_{3} \\ \alpha_{2} r_{2} \cos \theta_{2} & r_{2} \omega_{2}^{2} \sin \theta_{2} & -r_{3} \omega_{3}^{2} \cos \theta_{3}\end{array}\right]$
We proceed with the analysis, starting with the free-body diagram of each individual link.

Figure 4. The Schematic diagram of the Offset Slider Crank
Figure shows a schematic diagram of the offset slider crank with torque, $\tau_{12}$ applied to the crank, link 2 is the crank, link 3 is the connecting rod and link 4 is the slider block C which is free of all external loads. Now, by
applying Newton's law of motion to each link aided by free-body diagrams to figure 4, we can obtain our forces equations.

Applying equations of motion to link 2 gives:


Figure 5. Free-body diagram of link 2.
$F_{12, x}+F_{32, x}=0$ $\qquad$
$F_{12, y}+F_{32, y}=0$ $\qquad$
$-F_{12, x} r_{2} \sin \theta_{2}+F_{32, x} \cos \theta_{2}+\tau_{12}=0$ $\qquad$

For link 3, the free-body diagram leads directly to the equations as shown below:


Figure 6. Free-body diagram of Link 3

$$
\begin{align*}
& F_{32, x}+F_{43, x}=M_{3} A_{C_{3, x}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text { (14) }  \tag{14}\\
& F_{32, y}+F_{43, y}=M_{3} A_{C_{3, y}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text { (15) }  \tag{15}\\
& -F_{43, x}\left(r_{3}-r_{c_{3}}\right) \sin \theta_{3}+F_{43, x}\left(r_{3}-r_{c_{3}}\right) \cos \theta_{3}-F_{32, x} r_{C_{3}} \cos \theta_{3}+ \\
& F_{32, x} r_{C_{3}} \sin \theta_{3}=I_{3} \alpha_{3} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots \tag{16}
\end{align*}
$$

Applying equations of motion to link 4:


Figure 7. Free-body diagram of link 4

Link 4 is free of all external loads and analysis considers frictionless interface between the link 4 and the fixed link, hence, only one equation is obtained.
$F_{43, y}+F_{14, y}=0$
Therefore, the following forces are unknown:

$$
\left[F_{12, x} F_{12, y} F_{32, x} F_{32, y} F_{43, x} F_{43, y} F_{14, y} \tau_{12}\right]
$$

### 3.2. Centre of Mass (COM) Accelerations:

Since, the centre of mass (COM) of the crank $A B$ is on the rotational axis through A , it is considered stationary and all its accelerations are zeros.

Therefore, the centre of mass accelerations for link 3 are:
$A_{C_{3, x}}=-\alpha_{2} r_{2} \operatorname{Sin} \theta_{2}-r_{2} \omega_{2}^{2} \operatorname{Cos} \theta_{2}-\alpha_{3} r_{C_{3}} \operatorname{Sin} \theta_{3}-r_{C_{3}} \omega_{3}^{2} \operatorname{Cos} \theta_{3}$
$A_{C_{3, y}}=\alpha_{2} r_{2} \operatorname{Cos} \theta_{2}-r_{2} \omega_{2}^{2} \operatorname{Sin} \theta_{2}+\alpha_{3} r_{C_{3}} \operatorname{Cos} \theta_{3}-r_{C_{3}} \omega_{3}^{2} \operatorname{Sin} \theta_{3}$

Hence, the following accelerations are introduced:

$$
\left[A_{C_{3, x}} A_{C_{3, y}} \ddot{r}_{1} \alpha_{3}\right]
$$

Equations (7) through (19) create a system of twelve linear equations that form the sparse matrix below:

Where: $M_{3}=$ Mass of connecting rod, $M_{4}=$ Mass of slider block, $r_{2}=$ Crank length,$r_{3}=$ Connecting rod length, $r_{C_{3}}=$ Position of COM of connecting rod, $S_{2}=\operatorname{Sin} \theta_{2}, C_{2}=\operatorname{Cos} \theta_{2}$, $S_{3}=\operatorname{Sin} \theta_{3}, C_{3}=\operatorname{Cos} \theta_{3}, I_{3}=$ Mass moment of inertia of connecting rod about $\mathrm{COM}, A_{C_{3, x / y}}=\mathrm{COM}$ acceleration of connecting rod, $\alpha_{3}=$ Angular acceleration of the connecting rod, $\tau_{12}=$ Torque on Crank, and $\ddot{r}_{1}=$ Acceleration of link 1.

### 4.0 METHODOLOGY

This paper demonstrates using MATLAB and
SIMULINK, a simulation model for the synthesised offset slider-crank mechanism.

The model simulation would include the following scopes displayed:

1. The reaction force magnitude/ time graph at pivots $A, B$ and $C$ and the normal reaction force/time exerted on sliderblock C from the guide way;
2. The displacement/time graph of slider-block C; and
3. The torque/time graph of driving torque on crank $A B$;

The offset slider-crank mechanism model has the following specifications:

1. Stroke length of slider block $C=80 \mathrm{~mm}$
2. Advance-to-return time ratio $=1.1$
3. Offset height $h=20 \mathrm{~mm}$.

Also, suppose that:

1. Crank $A B$ has a mass of 0.2 kg and a moment of inertia of $100 \mathrm{~kg} . \mathrm{mm}^{2}$ about a rotational axis through its centre of mass;
2. Connecting rod BC has a mass of 0.5 kg and a moment of inertia of $1200 \mathrm{~kg} \cdot \mathrm{~mm}^{2}$ about a rotational axis through its centre of mass;
3. The centre of mass of connecting rod BC is located half way between the two pivots, $B$ and $C$;
4. The centre of mass of crank $A B$ is on the rotational axis through A ;
5. Slider block C has a mass of 0.8 kg ;
6. Friction at the interfaces and weights of all links are negligible compared to the dynamic forces;
7. Slider-block $C$ is free of all external loads;
8. The starting position of crank $A B$ is $\theta=0^{\circ}$; and
9. Crank $A B$ is driven at a uniform rotating speed of 2400 rev/min anticlockwise.
4.1 The MATLAB m-function
functionxout=slrcrndy1(u)
\%
\% function [xout]=slrcrndyn1(u)
$\%$ function to implement the full dynamic simulation
\% of a slider crank
$\% \mathrm{u}(1)=$ Theta- 2
$\% \mathrm{u}(2)=$ Theta -3
$\% \mathrm{u}(3)=\mathrm{r}-1$
$\% u(4)=$ Omega-2
$\% u(5)=$ Omega-3
$\% \mathrm{u}(6)=\mathrm{r}-1-\mathrm{dot}$
$\% \mathrm{u}(7)=\mathrm{F}-\mathrm{ext}$
\% Define local variables
$\mathrm{r} 1=\mathrm{u}(3)$;
$\mathrm{r} 2=0.15$; \% crank length in metres
$r 3=0.35$; \% connecting rod length in metres
rc2 $=0.0$; \% COM at pivot, implying a balanced crank
$\mathrm{rc} 3=\mathrm{r} 3 / 3 ; \% \mathrm{COM}$ is $1 / 3$ of the distance from the pivot with the crank
$\mathrm{C} 2=\cos (\mathrm{u}(1)) ; \mathrm{S} 2=\sin (\mathrm{u}(1)) ;$
$\mathrm{C} 3=\cos (\mathrm{u}(2)) ; \mathrm{S} 3=\sin (\mathrm{u}(2))$;
$\mathrm{w} 2=\mathrm{u}(4) ; \mathrm{w} 3=\mathrm{u}(5)$;
Fext $=u(7)$;
\% Define inertial parameters
$\mathrm{M} 2=1 ; \%$ mass of crank in kg
$\mathrm{M} 3=0.2 ; \%$ mass of connecting rod in kg
$\mathrm{M} 4=0.3 ; \%$ mass of slider in kg
I3 $=0.01$; \% mass moment of inertia of connecting rod about COM in kg.m^2

## \%

$\mathrm{a}=\mathrm{zeros}(14)$;
$b=z e r o s(14,1) ;$
$a(1,1)=1 ; a(1,3)=1 ; a(1,11)=-M 2 ;$
$a(2,2)=1 ; a(2,4)=1 ; a(2,12)=-M 2 ;$
$a(3,3)=-r 2 * S 2 ; a(3,4)=r 2 * C 2 ; a(3,8)=1$;
$a(4,3)=-1 ; a(4,5)=1 ; a(4,13)=-M 3 ;$
$a(5,4)=-1 ; a(5,6)=1 ; a(5,14)=-M 3$;
$a(6,3)=-r c 3 * S 3 ; a(6,4)=r c 3 * C 3 ; a(6,5)=(r c 3-r 3)^{*} S 3 ; a(6,6)=(r 3-$
rc3)* $\mathrm{C} 3 ; \mathrm{a}(6,10)=-\mathrm{I} 3$;
$a(7,5)=1 ; a(7,9)=M 4 ;$
$a(8,6)=-1 ; a(8,7)=1 ;$
$a(9,9)=1 ; a(9,10)=r 3^{*} S 3 ;$
$a(10,10)=-r 3 * C 3$;
$a(11,11)=1$;
$a(12,12)=1 ;$
$\mathrm{a}(13,10)=\mathrm{rc}^{*} \mathrm{~S} 3 ; \mathrm{a}(13,13)=1$;
$\mathrm{a}(14,10)=-\mathrm{rc} 3^{*} \mathrm{C} 3 ; \mathrm{a}(14,14)=1$;
\%
$b(7)=$ Fext;
$b(9)=-r 2^{*} C 2^{*} w 2^{\wedge} 2-r 3^{*} C 3^{*} w 3^{\wedge} 2 ;$
$b(10)=-r 2^{*} S 2^{*} w 2^{\wedge} 2-r 3^{*} S 3^{*} w 3^{\wedge} 2 ;$
$b(11)=-\mathrm{rc}^{*}{ }^{*} \mathrm{C}^{*} \mathrm{w} 2^{\wedge} 2$;
$b(12)=-r c 2^{*} S 2^{*} w 2^{\wedge} 2 ;$
$\mathrm{b}(13)=-\mathrm{r} 2^{*} \mathrm{C} 2^{*} \mathrm{w} 2^{\wedge} 2-\mathrm{rc} 3^{*} \mathrm{C} 3^{*} \mathrm{w} 3^{\wedge} 2$;
$b(14)=-r 2^{*} S 2^{*} w 2^{\wedge} 2-r c 3^{*} S 3^{*} w 3^{\wedge} 2$;
$\%$
\% Solve the equation
$x=\operatorname{inv}(a)^{*} b ;$
\%
\% Compute consistency error
error $=\operatorname{norm}([r 1-r 2 * C 2-r 3 * C 3, r 2 * S 2+r 3 * S 3]) ;$ \%
\% Set up output vector
xout(1)=x(10); \% Alpha-3
$\operatorname{xout}(2)=x(9) ; \%$ r1-double-dot
xout(3) $=x(8) ; \%$ Torque
$\operatorname{xout}(4)=x(1) ; \%$ F12x
$\operatorname{xout}(5)=x(2) ; \%$ F12y
$\operatorname{xout}(6)=x(3) ; \%$ F32x
$\operatorname{xout}(7)=x(4) ; \%$ F32y
$\operatorname{xout}(8)=x(5) ; \%$ F43x
$\operatorname{xout}(9)=x(6) ; \%$ F43y
xout(10)=x(7); \% F14y
xout(11)=error; \% Consistency error

### 4.2 The Simulation Model

Figure 8 below shows the Offset Slider Crank simulation model:


Figure 8. Full Dynamic Simulation of Offset Slider Crank

The Simulink model has the crank as the input link which turns at a constant rate of 2400rpm (251rad/sec) providing a torque to the whole mechanism.

The mechanism model has five (5) integrators: One to integrate the crank speed, $\omega_{2}$ to crank angle, $\theta_{2}$; two more to integrate the accelerations, $\theta_{3}$ and $\ddot{r}_{1}$, and finally, two more to integrate the resulting velocities to displacements. The sparse matrix is solved using $m$-function which takes all the integrator outputs as arguments. The function computes the accelerations and forces shown in the resulting output graphs and performs consistency check to ensure that the formulation is error-free and that the integration routines are maintaining adequate accuracy.

The initial conditions set for the simulation are given in table1 below and the simulation time was 0.0682 seconds. Results displayed are: the reaction force magnitude-time graph at pivots $\mathrm{A}, \mathrm{B}$ and C and the normal reaction force-time graph exerted on the slider block c from guide way; the displacement-time graph of the slider block c; the torque-time graph and finally, the coupler curve of the mass of connecting rod.
4.3. Initial conditions set for each integrator in the model

The initial conditions are determine using MATLAB function called the comvel.m.

```
function x=compvel(u)
% function to compute the slider velocity
% and the angular velocity of the connecting rod
% given the angular crank velocity as input
%
r2=0.15;
r3=0.35;
%
w2=u(1);
theta2=u(2);
theta3=u(3);
%
a=[1 r3*}\operatorname{sin}(theta3); 0-r3* cos(theta3)]
b=[-w2*r2*}\operatorname{sin}(theta2); w2*r2* cos(theta2)]
x=inv(a)*
end
```

Table 1 shows the five initial conditions that were used for the simulation.

Table 1. Initial Conditions for the Offset Slider-crank Mechanism

| Variable | Initial <br> condition |
| :---: | :---: |
| $\theta_{2}$ | 0 rad |
| $\theta_{3}$ | 0.18117 rad |
|  |  |
| $\omega_{2}$ | 251.3274 |
|  | $\mathrm{rad} / \mathrm{s}$ |
| $\omega_{3}$ | -89.6565 |
|  | $\mathrm{rad} / \mathrm{s}$ |
| $\dot{r}_{1}$ | $1.7931 \mathrm{~m} / \mathrm{s}$ |
|  |  |

Figure 9 through 14 display plots from the simulation results.
4.4 Simulation Results


Figure 9. Slider Pin Reaction force graph


Figure 10. Slider Block Displacement/Time graph


Figure 11. Crank Pin Reaction force graph


Figure 12. Grid Reaction force graph


Figure 13. Driving torque on Crank Torque/Time graph


Figure 14. Guide way Normal Reaction Force/Time graph

### 5.0. CONCLUSION

In this simulation, simultaneous constraint method is employed. Equations derived from the kinematic and dynamic analyses are assembled into a system of twelve linear equations to obtain the sparse matrix. This is solved
by the $m$-file function in the simulation process and the simulation results displayed in form of graphs.

### 6.0 REFERENCES

1. Ambekar A. G (2007). Mechanism and Machine

Theory. Prentice-Hall of India Private Limited, New
Delhi, India. (Asoke K. Ghosh)
2. A. G. Erdman and G. S. Sandor, Mechanism Design

- Analysis and Synthesis, 3rd edn (Prentice Hall, Upper Saddle River, NJ, 1997).

3. J. E. Shigley and J. J. Uicker, Jr, Theory of Machines and Mechanisms, 2nd edn (McGraw-Hill, Toronto, 1995).
4. H. H. Mabie and C. F. Reinholtz, Mechanics and Dynamics of Machinery, 4th edn (John Wiley \& Sons, Toronto, 1987). [4] Canadian Engineering
5. Molian, S. (1997) Mechanism Design, Practical Kinematics and Dynamics of Machinery, Pergamon Press.
6. Norton, R. L. (1999) Design of Machinery. An

Introduction to Synthesis and Analysis of Mechanisms of Machines, 2nd Edition, McGrawHill International Editions.
7. S. Dutta and T. K. Naskar (2013). Synthesis of Adjustable Offset Slider-Crank Mechanism for Simultaneous Generation of Function and Path using Variable-Length Links. Proceedings of the 1st International and 16th National Conference on Machines and Mechanisms (iNaCoMM2013), IIT Roorkee, India, Dec 18-20 2013
8. Jung-Fa Hsieh, (2011). Design and Analysis of Offset

Slider-Crank with Translating Roller-Follower.
Transactions of the Canadian Society for
Mechanical Engineering, Vol. 35, No. 3, 2011.
9. Shrikant R. Patel and D.S.Patel, (2013). Dynamic

Analysis of Quick Return Mechanism Using MATLAB.

